This sketch should not close without a brief reference to the influence that M. Lemoine has exerted in the realm of music. The soirées of M. and Mme. Lemoine are justly celebrated, and each week of the winter sees an assemblage representing the *anciens élevées* of the École Polytechnique, the École Normale, the Marine, and in general a good part of the scientific, literary, and artistic circles of Paris, to listen to a musical programme as original as the mathematical labors of the host. These soirées have exerted a great influence in a musical way, the type which they have fixed being adopted by many societies in and about Paris. One amusing feature of these meetings is the name which designates them. If the writer may be pardoned a personal allusion, he once attended an examination in the École Polytechnique by M. Hermann Laurent. It was one of the most severe he had ever seen,—an exceptionally bright young man submitted to an oral examination that would certainly have floored most American professors,—the examiner, a dyspeptic looking man as cold and as keen as steel and apparently as unsympathetic as ice, though in reality one of the most genial of men. To this justly celebrated mathematician, M. Laurent, is due the name of M. Lemoine's soirées, "La Trompette." Long ago he one day remarked to M. Lemoine in a jesting way, as the latter was excusing himself to attend one of his musical reunions, "Stay here with me, let the trumpet alone." Struck by the name, Lemoine adopted it, and *La Trompette* has ever since designated the delightful soirées with which the Paris cultured world is familiar.

A final word concerning the modesty of M. Lemoine. He estimates his position exactly. He says that he is not a mathematician. He has no claim to rank with Hermite, Poincaré, Picard, Painlevé, Appell, Jordan, Bertrand, Tannery, Darboux, or any of that famous circle which is making Paris such a center of study in the fields of higher modern mathematics. But all mathematicians feel that he has done a noteworthy work in other lines, and for this his name will be known and prominently known in the history of mathematics.

Ypsilanti, Michigan, March, 1896.

WHERE MATHEMATICIANS ARE NEEDED.

By ERIC DOOLITTLE, A. M., Chicago, Illinois.

There is no study of which the conceptions are more grand, nor of which the theorems are more comprehensive and profound than the study of Physical Astronomy. There is no study affording an application of Pure Mathematics in which the perfect harmony of its various parts is more evident; none in which

reason plays a greater part nor approximation a less one. The beauty and simplicity of its first propositions richly reward the early attention of the student, and in the end he is led to the wonderful theorems of *La Place* on the stability of the solar system and the conditions of its formation; theorems which *Baron Fourier* has justly named the highest which the human intelligence can propose.

It is remarkable that more young mathematicians do not enter this absorbing field. The common impression that it requires an unusual mathematical training is largely erroneous. Such a thorough knowledge of Calculus and Mechanics as is shown by the many contributors of the MONTHLY is fully sufficient. Physical Astronomy demands patient and steadfast work; mere brilliance and versatility can accomplish no more of fundamental importance here than in any other true science.

I would urge upon those who are now fitted to enter this or other like work, the great necessity of concentrating their energies upon it. It should be the one object of every devoted student to perfect and advance his own science. It is to this that his whole work must be directed. To such an one years of fragmentary study, first on one subject and then on another, are utterly wasted.

It is the disastrous mistake of many students that they do not realize how soon study for mere amusement or culture should give place to something higher. They fear, often mistakenly, that they are incapable of beginning work of real importance: instead of arranging then a definite series of studies to prepare themselves, they continue to dissipate their strength and accomplish nothing.

Physical Astronomy is calling in many directions for original work. In this country it is comparatively neglected. There are many who are being attracted by the pleasures of Photography and Spectroscopy, but there are few who realize the field which the Fundamental Astronomy opens to them. It contains many problems of the deepest interest. It is filled with questions whose answer requires, not an expensive observatory, but rather mathematical patience or skill.

Readers of the MONTHLY who are determined to accomplish something may well devote themselves to this science. The certainty of their adding to the sum of human knowledge is here greater than in Pure Mathematics, the reward of faithful work unaccompanied by special genius far more certain. The explanation of the variable stars, of the cause and nature of the sun's peculiar rotation, more complete theories of the satellites and of the figures and attractions of the Heavenly Bodies, the determination of the perturbations of the asteroids and other planets and the causes of the anomalies which occur, and the able discussion of a multitude of observations relating to these and other problems are a very few of the many directions in which original work is needed.

As with any true science, Physical Astronomy requires from those who enter upon it long and patient devotion. Its rewards are not bestowed by

chance, nor are they on that account of less value. It is of little popular interest. Its discoveries are seldom sensational. But its dignity and importance cannot be over-estimated. Of American Astronomers, the names of Hill and Newcomb will go down through the ages: their researches will never lose their importance. And whoever adds to this science is contributing to a knowledge which shall endure forever.

Baron Fourier said of La Place:

"Your successors, gentlemen, will witness the accomplishment of the great phenomena whose laws he discovered. They will observe in the motions of the Moon the changes which he predicted and of which he alone was able to assign the cause. The continued observation of Jupiter's satellites will perpetuate the memory of the inventor of the laws which govern them in their courses. The great inequality of Jupiter and Saturn, running through their long periods, and giving to these bodies new situations will recall without ceasing one of his most astonishing discoveries. These are the titles of a true glory which nothing can extinguish. The spectacle of the heavens will be changed, but at those remote epochs the glory of the inventor will continue forever; the traces of his genius bear the seal of immortality."

Chicago University, February 9th, 1896.

NON-EUCLIDEAN GEOMETRY: HISTORICAL AND EXPOSITORY.

By GEORGE BRUCE HALSTED, A. M., (Princeton); Ph. D., (Johns Hopkins); Member of the London Mathematical Society; and Professor of Mathematics in the University of Texas, Austin, Texas.

[Continued from January Number.]

PROPOSITION XXII. *If two straights AB, CD existing in the same plane stand perpendicular to a certain straight BD; but AC joining these perpendiculars makes with them internal acute angles (in hypothesis of acute angle): I say (Fig. 26) the terminated straights AC, BD have a common perpendicular, and indeed within the limits fixed by the designated points A and C.*

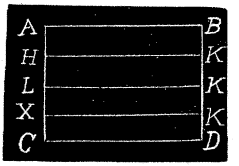

Fig. 26.

Proof. For if *AB*, *CD* are equal, it follows (from P. II) that the straight *LK*, by which these two *AC* and *BD* are bisected, will be to them a common perpendicular. But if either be the greater, as suppose *AB*; let fall to *BD* (according to Eu. I. 12) from any point *L* of *AC* the perpendicular *LK*, meet-